FREE DISCHARGE OF BULK MATERIALS THROUGH SINGLE-
AND MULTIPLE-HOLE BOTTOMS AND STATIONARY SPHERICAL PACKING
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Correlations are obtained for calculating the rate of flow of bulk materials through single- and multiple-hole bottoms and stationary spherical packing.

Single- and multiple-hole bottoms and granular stationary packings are structural. elements of many processing units (dryers, chemical reactors, mixers, etc.) in which dispersed media serve as the working substance.

A large number of investigations have by now been conducted on the free discharge of bulk substances through single openings [1-5, for example]. Only [6] has dealt with the free discharge of such media through a bottom with many openings. The latter presents partial data on the distribution of the rate of flow of particles of artificial graphite between separate holes in a multiply perforated hopper bottom. Correlations are recommended in [7-9] for calculating the flow rate of a bulk material in the case of its submerged discharge from multiple-hole bottoms into a mediun under a higher, and a lower, pressure than the unit. However, our analysis of these correlations shows that they contain misprints, making it impossible to use them for low gas filtration rates, i.e., under conditions close to free discharge. It should also be noted that the correlations generalize the results of tests in which the particle size and density, as well as the cross section of the holes in the bottom, were not changed. As far as we know, no published studies have been devoted to the free discharge of bulk materials in packings. Presented below are the results of a set of investigations on the movement of dispersed media under the influence of gravity through flat single- and multiple-hole bottoms and stationary spherical packings.

The structural elements through which the bulk material moved were located in vertical metal columns with a cross section that was constant along their height. Table 1 shows the main geometric characteristics of the columns and structural elements. Columns with a characteristic dimension $D=80$ and 150 mm were square in cross section, while the remaining columns were circular in cross section. Transparent windows of organic glass were installed in the column walls to permit observation of the movement of the dispersed medium. The height of the columns ranged from 600 to 1500 mm .

In the tests, we used dispersed materials with different physicomechanical properties and shapes and sizes of particles. However, all of the materials were characterized by good flowability. (Table 2). Materials $1-7$ were prepared on standard screens and consisted of several narrow fractions of particles. We took as the diameter of these particles the geo-metric-mean size of the mesh of adjacent screens encompassing the given fraction. The equivalent diameter $d_{h}$ was calculated as the weighted-mean diameter of the narrow fractions. The minimum and maximum fraction sizes of these materials fell within the range (0.3-1.5) dh. Materials $8-11$ consisted of spherical particles differing from each other in diameter by no more than $5 \%$. The value of dh for these materials was calculated as the arithmetic-mean diameter of a representative portion of the particles. The millet grains (material l2) were close to ellipsoidal in form. In the tests, we used only those grains that passed through screens between 1.75 and 2.05 mm in size.

The parameters $\rho b$ and tan $\alpha$ were determined by the standard methods described in [10].

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TABLE 1．Characteristics of the Investigated Systems

| No．of test series | D，mm | $d_{0}, \mathrm{~mm}$ | $\Omega$ | No．of bulk material from Table 2 |
| :---: | :---: | :---: | :---: | :---: |
| With single－hole bottom |  |  |  |  |
| 1 | 6，5－76，0 | 6，0－36，0 | 0，006－0，852 | 1－12 |
| 2 | 80，0 | 0，7－20，0 | Менее 0，050 | 1－12 |
| 3 | 150，0 | $24,0-40,0$ | －》－0，056 | $1-3,5,6,9,12$ |
| 4 | 300，0 | 40，0－60，0 | － | $1,3,6,10,12$ |
| With multiple－hole bottom |  |  |  |  |
| 5 | 80，0 | 2，9 | $\begin{array}{ll} 0,069 ; 0,097 ; & 0,149 ; \\ 0,288 ; 0,361 ; 0,499 \end{array}$ | $1-3,4,6,7$ |
| 6 | 80，0 | 6，0 | $\begin{aligned} & 0,0398 ; 0,0795 ; 0,172 ; \\ & 0,296 ; 0,415 ; 0,636 \end{aligned}$ | $1-3,5,6,8,11,$ |
| 7 | 80,0 | 8，2 | $\begin{aligned} & 0,0577 ; \\ & 0,533 \end{aligned}, 0,140 ; 0,371 ;$ | $1,3,5,6,9$ |
| 8 | 80，0 | 12，0 | 0，159；0，283；0，442； | 1，5－12 |
| 9 | 150，0 | 16，0 | 0，143；0，223； 0,321 | 1，4，6，12 |

With stationary spherical packing

|  |  | $d_{s}, \mathrm{~mm}$ | $\varepsilon$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 10 | 18，3 | 6，025 | 0，448 | 1，2， 6 |
| 11 | 26，3 | 6，025 | 0，455 | 1，2， 6 |
| 12 | 72，0 | 6，025 | 0，383 | 1， 5 |
| 13 | 80，0 | 6，025 | 0，362 | 1，2， 6 |
| 14 | 80，0 | 11，4 | 0，399 | $1-3,5,6$ |
| 15 | 80，0 | 15，3 | 0，420 | $1-6,10$ |
| 16 | 80,0 | 25，4 | 0，468 | $1-8,10$ |
| 17 | 80，0 | 25，4 | 0，501 | $1,3,6,8,11,1$ |
| 18 | 150，0 | 6，025 | 0.363 | 1， 6 |
| 19 | 150，0 | 11，4 | 0，376 | 1，3， 5 |

TABLE 2．Characteristics of the Bulk Materials

| Serial No． | Material | $d_{\text {a }} \mathrm{mm}$ | $\rho_{\mathrm{b}}, \mathrm{kg} / \mathrm{m}^{3}$ | $\operatorname{tg} \alpha$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Sand | 0，142 | 1430 | 0，637 |
| 2 | \＃ | 0，252 | 1450 | 0，630 |
| 3 | ＂ | 0，603 | 1700 | 0，687 |
| 4 | 》 | 0，748 | 1700 | 0，680 |
| 5 | Silica gel | 0，310 | 465 | 0，652 |
| 6 | Alundum | 0，147 | 1700 | 0，728 |
| 7 | Fireclay | 0，935 | 1186 | 0，743 |
| 8 | Glass beads | 1，120 | 1530 | 0，510 |
| 9 | 》 | 1，580 | 1530 | 0，520 |
|  | Shot |  |  |  |
| 10 | steel | 0,700 1,920 | 4340 6760 | 0,590 0,343 |
| 11 | $\mathrm{Millet}^{\text {lead }}$ | 1,920 1,900 | 6760 787 | 0,343 0,560 |
|  |  |  |  |  |

The experiments were conducted on a closed－type unit．A column with the structural element being studied was placed under a feed hopper which had previously been loaded with the bulk material．The material was admitted to the column by opening a gate in the hopper outlet pipe．The same gate was used to regulate the depth of the bed of free－flowing mate－ rial in the column during the test．Passing through the test element，the dispersed mate－ rial fell into a receiving hopper positioned under the column．The receiving hopper had two compartments．The material was directed into one of these compartments when the height of the moving bed was being adjusted or the flow in the column was being stabilized．The second compartment received the material when the required measurements were being made．

The spaces in the hopper and column under the beds of material were in contact with the atmosphere in all of the tests．Also，the shells of the column walls were grounded to pre－ vent the buildup of static electricity．

## TESTS ON SINGLE－AND MULTIPLE－HOLE BOTTOMS

The bottoms were made of steel sheets from 1.0 to 5.0 mm thick．The sheets were in－ stalled exactly horizontally in the columns．The holes in the bottom were cylindrical in


Fig. 1. Functions $G_{0} / G=f_{1}(\Omega)$ and $\bar{G}_{0} / G=f_{2}(\Omega)$ at $d_{0}=12.0 \mathrm{~mm}: 1-3$ ) experimental points (bulk materials 2,5 , and 11 , respectively - see Table 2); a) estimate from Eq. (2) ; b) estimate from Eq. (3).
form. The hole axis in the single-hole bottoms coincided with the column axis. The multihole bottoms were made in two sets. In one set the holes were staggered over the surface of the bottom, while in the other set they were arranged in rows. The ratio $\mathrm{S} / \mathrm{S}_{2}$ for the bottoms of the first set and $S_{1} / S_{2}$ for the bottoms of the second set was constant and was equal to 1.0. The distance from the axes of the outermost longitudinal and transverse rows of holes in the multihole bottoms to the column walls closest to these rows was equal to $S_{2} / 2$ and $S_{1} / 2$, respectively.

It was shown in [2] that the rate of discharge of bulk materials through a single-hole bottom is affected by the simplex $d_{0} / D$. The ratios $d_{0} / S_{1}, d_{0} / S_{2}$, and $d_{0} / S$ play an equivalent role in the case of multhole bottoms with randomly located holes. To compare the results of discharge of the bulk materials from the bottoms used in our experiments, it turned out that we could substitute for these simplexes a single general parameter $\Omega$. This parameter is related to the specific arrangement of the holes over the surface of the test bottoms.

The first factor whose effect on the gravity flow rate was determined was the height of the bed of bulk material in the column $h$. The tests showed that, for all of the investigated types of bottoms, the bed height had no effect on the flow rate. The sole exception here was the discharge of the materials when $h<2 d_{0}$. This result agrees with the data obtained in [1-5] in tests of the discharge of dispersed materials through single-hole bottoms.

In series $1,2,5$, and 6 we established the value of the parameter ( $d_{o} / d_{h}$ ) cr, below which discharge of the material from the bottom ceases. The experiments showed that the value of $\left(d_{0} / d_{h}\right)_{c r}=4.5-4.8$ for single-hole bottoms and depends on the form of the particles and the degree of polydispersity of the bulk material. For multihole bottoms, ( $d_{0} / d_{h}$ ) cr also depends on the parameter $\Omega$. Thus, at $\Omega \leqslant 0.07$, the value of $\left(d_{0} / d_{h}\right)_{c r}=4.5-4.8$, i.e., the same as for single-hole bottoms. At $0.07<\Omega \leqslant 0.636$, the value of the ratio rapidly decreases to the level 3.2-3.7. This drop in flow rate, characteristic of multihole bottoms, occurs as follows. When the value of $d_{o} / d_{h}$ decreases to values below 4.5-4.8, the particles first jam up over one or several of the holes. If $\Omega \leqslant 0.07$ here, then the relative spacings between the holes $S_{1} / d_{0}$ and $S_{2} / d_{0}$ are not great enough so that the vibrations of the particles in the dynamic roofs "working" at a given moment cause the static roofs over the obstructed holes to crumble. In this case, the process of hole blockage proceeds rapidly and discharge from the bottom ceases. When the parameter $\Omega>0.07$, the process of fracture of the static roofs becomes feasible and the obstructed holes can again begin to "work." The tests showed that when the bulk material is discharged at ( $3.2-3.7$ ) < $\mathrm{d}_{0} / \mathrm{d}_{\mathrm{h}}<$ (4.5-4.8), then at each moment of time the fraction of "working" holes $N<1$. The value of N decreases with a decrease in $\mathrm{d}_{0} / \mathrm{d}_{\mathrm{h}}$ and is roughly $0.75-0.8$ at values of $\mathrm{d}_{0} / \mathrm{dh}_{\mathrm{h}}$ close to (3.2-3.7).

In series $1-9$ at $d_{0} / d_{h}>(4.5-4.8)$, the bulk-material flow rate varied broadly within a wide range of values of parameter $\Omega$. Figure 1 shows only some of the experimental points, which clearly demonstrate the effect of $\Omega$ on the relative bulk-material flow rates $G_{0} / G$ and $\bar{G}_{0} / G$. It follows from the data shown that the value of $\Omega=0.07$ divides the graphs into two characteristic regions. At $\Omega \leqslant 0.07$, the values of $G_{0} / G$ and $\bar{G}_{0} / G$ are independent of $\Omega$ and are numerically equal to 1.0 . In the region $\Omega>0.07$, the values of $G_{0} / G$ and $\bar{G}_{0} / G$ are functions of both the parameter $\Omega$ and other_characteristics of the investigated systems. The value of $G$, which enters into $G_{0} / G$ and $\bar{G}_{0} / G$, depends on three parameters: $d_{0}$, $d_{h}$, and $\rho_{b}$.


Fig. 2. Dependence of dimensionless rate of flow of bulk materials through single-hole bottoms on relative diameter of the hole $d_{0} / d_{h}: 1-12$ ) experimental points (bulk materials l-12 from Table 2 ) ; curve - calculated from Eq. (1).

This follows both from [2-4] and our measurements. Figure 2 shows the graph of this dependence in dimensionless form. In this figure, the curve which generalizes our experimental data corresponds to the empirical formula

$$
\begin{equation*}
\operatorname{Fr}=1.06\left[1-1.2\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right)^{0.4}\right]^{-2.5} \tag{1}
\end{equation*}
$$

This formula was obtained for the interval (4.5-4.8) $<\mathrm{d}_{0} / \mathrm{d}_{\mathrm{h}}<432$. The mean-square scatter of the experimental points relative to the curve generalizing these points is $\pm 6.2 \%$. Since $\operatorname{Fr} \sim I / G^{2}$, then the mean-square error in the determination of $G$ from (1) does not exceed $2.5 \%$. Obviously, the correlation obtained can also be used to calculate $G_{0}$, $\bar{G}_{0}$, and $\mathrm{G}_{\mathrm{m}}$ if the parameter $\Omega \leqslant 0.07$. In these cases, the quantity $G$, entering into Fr , is replaced by $G_{0}, \bar{G}_{0}$, and $G_{m} / n_{0}$, respectively.

Equation (1) agrees satisfactorily with the relations proposed in [2-4]. However, the scatter of the empirical data in our experiments relative to (1) is less than in these works.

There have been studies [5, etc.] in which the quantity tan $\alpha$ has been introduced to account for dry friction in the correlations for calculating $G$. The test data shown in Fig. 2 indicate, that $G$ is independent of $\tan \alpha$. According to the results of our experiments, this is also true of $G_{0}$ and $\bar{G}_{0}$ if $\Omega \leqslant 0.07$.

Analysis of the original data, including the tests shown in Fig. 1 , showed that the individual properties of the bulk materials and their associated gas phase are manifest in the range of values $0.07<\Omega<1.0$. Accounting for these properties required the introduction of several parameters - tan $\alpha$, the number Ga , and $\rho \mathrm{g} / \mathrm{\rho b}$. It was also established that the rate of flow of the bulk material within the above range of $\Omega$ is affected by the geometric simplexes $\Omega$ and $d_{h} / d_{o}$, as well as by the simplex $d_{h} / D$ in the case of single-hole bottoms.

The completed study resulted in the following generalizing relations for the empirical data in the range $0.07<\Omega<1$ :
for single-hole bottoms

$$
\begin{equation*}
\frac{G_{0}}{G}=1+12.7\left(\frac{\Omega}{\operatorname{tg}^{2} \alpha}-2.2\right)(\Omega-0.07)\left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{b}}}\right)^{0.48} \mathrm{Ga}^{-0,08}\left[1+13\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right)^{2}\right]\left(1+5 \frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right) \tag{2}
\end{equation*}
$$

for multihole bottoms

$$
\begin{equation*}
\frac{\bar{G}_{0}}{G}=1+26\left(\frac{\Omega}{\operatorname{tg}^{2} \alpha}-1.7\right)(\Omega-0.07)\left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{b}}}\right)^{0.48} \mathrm{Ga}^{-0.08}\left[1+13\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right)^{2}\right] \tag{3}
\end{equation*}
$$

The exponents and proportionality factors with the multipliers in the right sides of Eqs. (2) and (3) were computed on a computer by the least-squares method. Figure 3 compares the calculated and experimental data. The relative flow rates $G_{0} / G$ and $\bar{G}_{0} / G$ calculated from


Fig. 3. Comparison of experimental and calculated data: 112) experimental points (bulk materials 1-12 from Table 2); curves I and II) calculated from Eqs. (2) and (3).

Eqs. (2) and (3) are plotted off the $x$ axis, while the corresponding values obtained in the experiments $\left(G_{0}\right)$ ex $/(G)$ ex and $\left(G_{0}\right)$ ex/ $(G)$ ex are plotted off the $y$ axis. The mean scatter of the empirical data is $\pm 6.1 \%$ relative to (2) and $\pm 5.6 \%$ relative to (3).

Allowing for (1), we convert Eqs. (2) and (3) to the form in which they will be used below to study discharge of the bulk material from the packing:

$$
\begin{gather*}
\operatorname{Fr}_{0}=1,06\left[1-1,2\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right)^{0.4}\right]^{-2.5}\left\{\left\{1+12,7\left(\frac{\Omega}{\operatorname{tg}^{2} \alpha}-2.2\right)(\Omega-0.07)\left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{b}}}\right)^{0.48} \mathrm{Ga}^{-0.08} \times\right.\right. \\
\left.\left.\times\left[1+13\left(\frac{d_{\mathrm{h}}}{d_{0}}\right)^{2}\right]\left(1+5 \frac{d_{\mathrm{h}}}{D}\right)\right\}^{2}\right\}^{-1}  \tag{4}\\
\mathrm{Fr}_{\mathrm{m}}=1.06\left[1-1,2\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right)^{0,4}\right]^{-2.5}\left\{\left\{1+26\left(\frac{\Omega}{\operatorname{tg}^{2} \alpha}-1,7\right)(\Omega-0,07)\left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{b}}}\right)^{0.48} \mathrm{Ga}^{-0,08} \times\right.\right. \\
\left.\left.\times\left[1+13\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{o}}}\right)^{2}\right]\right\}^{2}\right\}^{-1} \tag{5}
\end{gather*}
$$

Equations (2)-(5) were obtained for the following ranges of the variables: $18^{\circ} 55^{\prime} \leqslant$ $\alpha \leqslant 36^{\circ} 40^{\prime} ; 361 \leqslant \rho_{b} / \rho_{\mathrm{g}} \leqslant 5600 ; 0.12 \leqslant \mathrm{Ga} \leqslant 309$. It should be noted that they are valid at $0.07 \leqslant \Omega \leqslant 0.852$ and $(4.5-4.8) \leqslant d_{o} / d_{h} \leqslant 252$ for single-hole bottoms and 0.07 $\Omega \leqslant 0.636$ and $(4.5-4.8)<d_{o} / d_{h}<112$ for multihole bottoms. In the latter case, the number of holes in the bottom should be no less than nine.

## TESTS ON SPHERICAL PACKINGS

Stationary packings are fillings (for columns) of smooth steel balls of monodispersed composition. The support for the spheres in our tests was provided by metal-wire nets secured to the base of the columns. The height of the packings in the tests was (1-20) D. The mean (over the volume) separateness of the packings $\varepsilon$ in series $10-13$ and 18 was calculated on the basis of measurements of the bulk and true densities of the spheres, while the same quantity was calculated for series $14-17$ and 19 from the volume of water poured into the spaces in the packing.

The supporting nets were chosen so that they did not limit the flow of the bulk material through the packing. The tests showed that this condition was met if the cross sectional area of the netting was not less than $\varepsilon$ and if the inside linear dimensions of its mesh fell within the range ( $0.2-0.6$ ) $\mathrm{d}_{\mathrm{s}}$.

To analyze the test data on the free discharge of bulk materials from spherical packing, the latter was represented by a system of curved "channels" of variable cross section along their height (capillary model of granular packing [11]). It is known [11] that in determining the rate of flow of continuous media in such a model, the mean separateness and


Fig. 4. Comparison of experimental data with calculated data: 1-11) experimental data for ( $G_{p}$ ) ex, $\mathrm{kg} / \mathrm{sec}$ (bulk materials $1-8$ and $10-$ 12 in Table 2); curve) estimate of $\mathrm{Gp}, \mathrm{kg} / \mathrm{sec}$, from Eq. (10).
the equivalent diameter of the "channels" deq are taken as the basic geometric parameters of the packing. For a packing comprised of spheres of the same diameter and located in square or circular columns, the value of deq is calculated from the formula

$$
\begin{equation*}
d_{\mathrm{eq}}=\frac{4 \varepsilon}{6(1-\varepsilon) / d_{\mathrm{s}}+k_{1} 4 / D} . \tag{6}
\end{equation*}
$$

The coefficient $k_{1}$ in (6) is determined by experiment. In particular, in studies devoted to measuring the fluid resistance of packings of identical-diameter spheres to gas [12] and dust-gas [13] flows, it was shown that the best fitis obtained between the experimental data and the curve generalizing this data at $k_{1}=0.75$. However, study of the free discharge of bulk materials from packings showed that in this case the movement of the dispersed material is determined by the local parameters of the "channels" in those sections which limit the rate of flow of the dispersed particles, not by the average parameters. Contact is broken between the particles in these sections, i.e., the dense, moving gravity bed becomes a falling flow. This is readily seen in the "channels" adjacent to the transparent windows in the column. However, it is impossible to directly determine the dimensions of the limiting sections.

Considering the above feature of the system being investigated, we introduced the effective cross section, defines as follows, as a geometric characteristic of the system's packing

$$
\begin{equation*}
\Omega_{\mathrm{ef}}=k_{2} \varepsilon \tag{7}
\end{equation*}
$$

along with the effective diameter of the limiting section of the "channels" def. The value of def was calculated from the following formula, by analogy with Eq. (6)

$$
\begin{equation*}
d_{\mathrm{ef}}=\frac{4 \Omega_{\mathrm{ef}}}{6\left(1-\Omega_{\mathrm{ef}}\right) / d_{\mathrm{s}}+k_{1} 4 / D} . \tag{8}
\end{equation*}
$$

The coefficient $k_{1}$ was determined on the basis of measurements of the rate of flow of sand $\mathrm{d}_{\mathrm{h}}=0.142 \mathrm{~mm}$ in series 13 and 18 . The systems in these tests differed only in the cross sections of the columns. It turned out that the column diameter does not affect the bulkmaterial flow rate per unit of cross-sectional area of the packing. In connection with this, we took zero for the value of $k_{1}$. Allowing for (7), Eq. (8) then takes the form

$$
\begin{equation*}
d_{\mathrm{ef}}=\frac{2}{3} \frac{k_{2} \varepsilon d_{\mathrm{s}}}{\left(1-k_{2} \varepsilon\right)} \tag{9}
\end{equation*}
$$

To determine the value of $\mathrm{k}_{2}$ and construct a correlation generalizing the test data on bulk-material flow rate in the packing, it was suggested that the functional relationship
between the parameters determining the process of discharge in the investigated system should be similar to the relationships defined by Eq. (4) or Eq. (5). We tried Eq. (5) first. Of course, here the geometric parameters $d_{0}$ and $\Omega$ and the quantity $G_{m} / n_{0}$ in (5) were replaced by def, $\Omega_{e f,}$ and $G_{p} / n_{p}$. At the same time, in view of the features of the system with a packing, some of the constants in the denominator of the right side of Eq. (5) were replaced by unknown coefficients subject to a new determination. As a result, the initial form of the dimensionless generalizing correlation appeared as the expression

$$
\begin{gather*}
\operatorname{Fr}_{\mathrm{p}}=1.06\left[1-1.2\left(d_{\mathrm{h}}^{\prime} d_{\mathrm{ef}}\right)^{0.4}\right]^{-2.5}\left\{\left\{1+k_{3}\left(\frac{\Omega_{\mathrm{ef}}}{\operatorname{tg}^{2} \alpha}-1.7\right)\left(\Omega_{\mathrm{ef}}-0.07\right)\left(\frac{\rho_{\mathrm{g}}}{\rho_{\mathrm{b}}}\right)^{m} \mathrm{Ga}^{-0.08 \times}\right.\right. \\
 \tag{10}\\
\left.\left.\times\left[1+13\left(\frac{d_{\mathrm{h}}}{d_{\mathrm{ef}}}\right)^{2}\right]\right\}^{2}\right\}^{-1}
\end{gather*}
$$

where $k_{3}$ and $m$, as well as $k_{2}$ in the parameters $d_{e f}$ and $\Omega e f$, are empirical coefficients determined on the basis of the best agreement obtained between (10) and the results of the tests in series 9-19. Computer selection yielded the following values

$$
\begin{equation*}
k_{3}=1.54, \quad m=0.08, \quad k_{2}=0.63 \tag{11}
\end{equation*}
$$

Considering (11), the combination $\left(\rho_{\mathrm{g}} / \rho_{\mathrm{b}}\right)^{0.08} \mathrm{Ga}^{-0.08}$ in Eq. (10) can be replaced by the complex $\mathrm{Ar}^{-0.0 \mathrm{~B}}=\left[\mathrm{Ga}\left(\rho_{\mathrm{b}} / \rho_{\mathrm{g}}\right)\right]^{-0.0 \mathrm{~B}}$.

Figure 4 shows empirical values of the rate of flow of a bulk material through packing $\left(G_{p}\right)$ ex in comparison with the calculated values $G p$ obtained from Eq. (10) with allowance for (11). The standard deviation of the empirical points from the theoretical points is 9.1\%. The formula was obtained for the following ranges of variables: $4.12<\mathrm{d}_{\mathrm{f}} \mathrm{f} / \mathrm{dh}<55$; $0.362 \leqslant \varepsilon \leqslant 0.501 ; 361<\rho_{\mathrm{b}} / \rho_{\mathrm{g}}<5600$; $18^{\circ} 55^{\prime}<\alpha<36^{\circ} 40^{\prime} ; 0.12<\mathrm{Ga}<309$. It should be noted that the standard deviation increases to $11.5 \%$ when the same empirical points are approximated with Eq. (4).

In all of the tests involving the discharge of bulk materials from packings, it was observed that the limiting sections in the channels adjacent to the column walls were located in the top part of the packing, (0.5-3.5) $\mathrm{d}_{\mathrm{s}}$ from its upper surface. It may be assumed that the limiting sections in most of the "channels" inside the packing are also located in this region. This assumption was borne out in tests in which the density $\rho_{V}$ of a bulk material moving in the channels of a packing was determined. The results of these tests are of interest in their own right.

The value of $\rho_{V}$ was measured as follows. We closed the gate on the feed hopper while the bulk material was being discharged from the system. We then used a device to quickly level out the free surface of the dispersed material moving in the column. At the moment the material reached the top surface of the packing, we slammed shut a gate located under the netting. We then computed the value of $\rho_{v}$ from the weight of the bulk material left in the system $M$ and the known volume of the cavities in the packing $V$ :

$$
\begin{equation*}
\rho_{\mathrm{v}}=M / V \tag{12}
\end{equation*}
$$

These tests showed that $\rho_{V}$ is independent of the height of the packing $H$ if $H>$ (1215) $\mathrm{d}_{\mathrm{S}}$. It also turned out that at $(12-15) \mathrm{d}_{\mathrm{S}}<\mathrm{H}<120 \mathrm{~d}_{\mathrm{s}}$ and $4.8<\mathrm{d}_{\mathrm{ef}} / \mathrm{d}_{\mathrm{h}}<55$, the value of $\rho v$ can be represented to within $\pm 7 \%$ for different bulk materials (Table 2) by a single relation

$$
\begin{equation*}
\rho_{\mathrm{v}}=0.6 \rho_{\mathrm{b}} \tag{13}
\end{equation*}
$$

When the height of the packing is less than $(12-15) d_{S}, \rho v i n c r e a s e s$ and approaches the value of $\rho b$. The results obtained here confirm the hypothesis on the location of the limiting sections in the top region of spherical packing, since $\rho_{V} \approx \rho_{b}$ above these sections. However, this does not mean that the separateness of the packing is lowest in this region. Measurements of separateness along the height of the packing showed that the lowest values are encountered in regions located considerably below the top region.

## NOTATION

Ga $\equiv \mathrm{gd}_{\mathrm{h}}^{3} / \nu^{2}$, Galileo number; D, diameter (side dimension) of column; do, diameter of hole(s) in bottom; $d_{h}$, equivalent diameter of particles of bulk material; $d_{s}$, diameter of packing spheres; deq, equivalent diameter of "channels" in packing; def, effective diameter
of limiting section of packing "channels"; F , column cross-sectional area; $\mathrm{Fr} \equiv(\pi / 4)^{2} \mathrm{gd}_{5}^{5} \rho_{\mathrm{b}}{ }^{2} / \mathrm{G}^{2}$, $\mathrm{Fr}_{0} \equiv(\pi / 4)^{2} \mathrm{gd}_{\mathrm{o}}^{5} \rho_{\mathrm{b}}^{2} / \mathrm{G}_{\mathrm{o}}^{2}, \quad \mathrm{Fr}_{\mathrm{m}} \equiv(\pi / 4)^{2} \mathrm{gd}_{\mathrm{o}}^{5} \rho_{\mathrm{b}}^{2} /\left(\mathrm{G}_{\mathrm{m}} / \mathrm{n}_{\mathrm{o}}\right)^{2}, \mathrm{Fr}_{\mathrm{p}} \equiv(\pi / 4)^{2} \mathrm{gd}_{\mathrm{e}}^{5} \rho_{\mathrm{b}}^{2} /\left(\mathrm{G}_{\mathrm{p}} / \mathrm{n}_{\mathrm{p}}\right)^{2}$ - Froude numbers; G, $G_{0}$, rates of flow of bulk material through a single-hole bottom at $\Omega \rightarrow 0$ and $\Omega>0$, respectively; $\bar{G}_{0}=\mathrm{G}_{\mathrm{m}} / \mathrm{n}_{0}$, rate of flow of bulk material per hole in a multihole bottom; $\mathrm{G}_{\mathrm{m}}$, rate of flow of bulk material through a multihole bottom; $\mathrm{Gp}_{\mathrm{p}}$, rate of flow of bulk material through a spherical packing; g, acceleration due to gravity; $H$, height of stationary spherical packing; $h$, height of moving bed of bulk material; $n_{0}$, number of holes in bottom; $n_{p}=$ $\mathrm{F}_{\mathrm{e}} \mathrm{f} /(\pi / 4) \mathrm{d}_{\mathrm{ef}}^{2}$, number of "channels" in packing; $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}$, longitudinal, lateral, and diagonal spacings of holes in multihole bottom, respectively; $\alpha$, angle of repose; $\rho_{\mathrm{v}}$, density of bulk material moving in "channels" in the packing; $\varepsilon$, mean (over the column) separateness of the packing; $\nu$, kinematic viscosity of the gas; $\rho g, \rho b$, density of gas and bulk density of bulk material, respectively; $\Omega=(\pi / 4) \mathrm{d}_{0}^{2} \mathrm{n}_{0} / \mathrm{F}$, $\Omega_{\text {ef }}$, cross section of the holes in the bottom and the effective cross section of the packing "channels" [see (7)]. Subscripts: cr, critical; ex, experimental.

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